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A Localization Property of Line Spectrum Frequencies

G. A. Mian and G. Riccardi

Abstract—**The interlacing property for the line spectrum frequencies (LSF's) is extended to the LSF's associated to successive order predictor polynomials. The corresponding separation theorem gives the most precise lower and upper bounds of the intervals which the LSF's may belong to.**

I. Introduction

Among the most used spectral parametrization parameters in speech coding, LSF's have many properties which make them suitable for speech encoding. They were first introduced in [1]. In [2]–[4] it is shown that LSF's encode speech spectral information more efficiently than other transmission parameters. Their efficient computation is addressed to in [5], [6]. Alternative interpretations of their role are given in [7], [8].

The starting point for deriving LSF's is the prediction error filter $A_M(z)$

$$A_M(z) = \sum_{k=0}^{M} a_k\, z^{-k}, \qquad a_0 = 1 \tag{1}$$

with M the predictor order and a_k the predictor coefficients which are the solutions to the Yule-Walker equations [9]. The intimate correspondence between LSF's, predictor taps and reflection coefficients has been shown with different approaches. The Levinson-Durbin solution of the Yule-Walker equations, concisely expresses a recursive relationship between the reflection coefficients, k_m, and the mth and $(m-1)$th order prediction filters $A_m(z)$ and $A_{m-1}(z)$

$$A_m(z) = A_{m-1}(z) + k_m z^{-1}\hat{A}_{m-1}(z) \tag{2a}$$

$$\hat{A}_m(z) = k_m A_{m-1}(z) + z^{-1}\hat{A}_{m-1}(z) \qquad m = 1, 2, \cdots, M \tag{2b}$$

with $\hat{A}_{m-1}(z) = z^{-m+1} A_{m-1}(z^{-1})$, the reciprocal polynomial of $A_{m-1}(z)$. By letting $k_m = \pm 1$ in (2a), we obtain two polynomials $P_m(z)$ $(k_m = 1)$ and $Q_m(z)$ $(k_m = -1)$, symmetric and antisymmetric, respectively. The LSF's corresponding to $(m-1)$th order polynomial $A_{m-1}(z)$, are the zeros of polynomials $P_m(z)$ and $Q_m(z)$

$$P_m(z) = A_{m-1}(z) + z^{-1}\hat{A}_{m-1}(z) = \sum_{i=0}^{m} p_i z^{-i} \tag{3a}$$

$$Q_m(z) = A_{m-1}(z) - z^{-1}\hat{A}_{m-1}(z) = \sum_{i=0}^{m} q_i z^{-i}. \tag{3b}$$

From (3a) and (3b) it is

$$A_{m-1}(z) = \frac{P_m(z) + Q_m(z)}{2} \tag{4a}$$

$$z^{-1}\hat{A}_{m-1}(z) = \frac{P_m(z) - Q_m(z)}{2} \tag{4b}$$

which give polynomials $A_{m-1}(z)$ and $z^{-1}\hat{A}_{m-1}(z)$ as linear combinations of the symmetric and antisymmetric polynomials $P_m(z)$ and $Q_m(z)$.

The main properties of LSF's and their relationships with the predictor coefficients have been studied in [1], [2] and in [7], [8]. A key result is that for $A_{m-1}(z)$ to be stable it is necessary and sufficient that the zeros of $P_m(z)$ and $Q_m(z)$ are simple, lie on the unit circle, interlace each other and $\text{sign}(p_0) = \text{sign}(q_0)$.

It is worth noticing that such a theorem holds for any couple of symmetric and antisymmetric polynomials obtained via $A_{m-1}(z) \pm z^{-l}A_{m-1}(z^{-1})$ for $l \geq 0$. The case $l = 0$ was dealt with in [10], the case $l = m-1$ in [11], [12] and the case $l = m$ corresponds to the polynomials in (3).

II. The Localization Property

Equations (2) and (3) allow one to find the relationship between the LSF's of two successive order polynomials, namely, between (P_m, Q_m) and (P_{m-1}, Q_{m-1}). Replacing in (3) the polynomial A_{m-1} with its expression in terms of (P_{m-1}, Q_{m-1}), we have

$$2P_m(z) = (1 + k_{m-1})(1 + z^{-1})P_{m-1} + (1 - k_{m-1})(1 - z^{-1})Q_{m-1} \tag{5a}$$

$$2Q_m(z) = (1 + k_{m-1})(1 - z^{-1})P_{m-1} + (1 - k_{m-1})(1 + z^{-1})Q_{m-1}. \tag{5b}$$

This recursive relationships lead to the following property for the LSF's of a stable predictor:

The LSF's of the mth order lie in the open intervals between the LSF's of the $(m-1)$th order.

More precisely, denoting with $\lambda_{m-1,i}$ and $\delta_{m-1,i}$, $i = 1, 2, \cdots$, the arguments of zeros of $P_{m-1}(z)$ and $Q_{m-1}(z)$ in the upper-half unit circle, respectively, and denoting with $\lambda_{m,i}$ and $\delta_{m,i}$ the same quantities of $P_m(z)$ and $Q_m(z)$, the $\lambda_{m,i}$ lie in the intervals $(\delta_{m-1,i}, \lambda_{m-1,i})$, $i = 1, 2, \cdots$, and the $\delta_{m,i}$ lie in the complementary intervals $(\lambda_{m-1,i-1}, \delta_{m-1,i})$, $i = 2, 3, \cdots$. An example is depicted in Fig. 1, for $m = 8$.

Proof: We shall refer to (5a) since similar reasonings apply to (5b). The symmetric and antisymmetric polynomials $P_m(z)$ and $Q_m(z)$ evaluated on the unit circle can be written as

$$P_m(e^{j\lambda}) = e^{-j\lambda m/2}\tilde{P}_m(\lambda) \tag{6a}$$

$$Q_m(e^{j\lambda}) = je^{-j\lambda m/2}\tilde{Q}_m(\lambda) \tag{6b}$$

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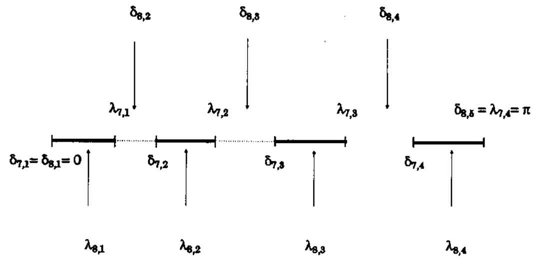

Fig. 1. Intervals containing the mth-order LSF's $\lambda_{m,i}$ and $\delta_{m,i}$ in the case $m = 8$.

where $\tilde{P}_m(\lambda)$ and $\tilde{Q}_m(\lambda)$ are real trigonometric polynomials [13, pp. 257–258]. Polynomials $\tilde{P}_m(\lambda)$ and $\tilde{Q}_m(\lambda)$ can be factorized as

$$m \text{ even} \begin{cases} \tilde{P}_m(\lambda) = 2^{m/2} \prod_{i=1}^{\frac{m}{2}} (\cos\lambda - \cos\lambda_{m,i}) \\ \tilde{Q}_m(\lambda) = 2^{m/2} \sin\lambda \prod_{i=2}^{\frac{m}{2}} (\cos\lambda - \cos\delta_{m,i}) \end{cases} \tag{7a}$$

$$m \text{ odd} \begin{cases} \tilde{P}_m(\lambda) = 2^{(m+1)/2} \cos\lambda/2 \prod_{i=1}^{\frac{m-1}{2}} (\cos\lambda - \cos\lambda_{m,i}) \\ \tilde{Q}_m(\lambda) = 2^{(m+1)/2} \sin\lambda/2 \prod_{i=2}^{\frac{m+1}{2}} (\cos\lambda - \cos\delta_{m,i}) \end{cases} \tag{7b}$$

where the $\lambda_{m,i}$ (and $\lambda = \pi$ for m odd) and the $\delta_{m,i}$ with $\delta_{m,1} = 0$ (and $\lambda = \pi$ for m even) are the LSF's corresponding to $P_m(z)$ and $Q_m(z)$, respectively. They are placed on the unit circle $z = e^{j\lambda}$ with $0 \le \lambda \le \pi$

$$\delta_{m,1} = 0 < \lambda_{m,1} < \delta_{m,2} < \lambda_{m,2} < \cdots \le \pi. \tag{8}$$

As far as (5a) is concerned, we consider the case m odd because the case m even can be dealt with along the same lines. The substitution of $\tilde{P}_{m-1}(\lambda)$ given by (7a) in (5a) leads to the relationship between $\tilde{P}_m(\lambda)$ and $(\tilde{P}_{m-1}(\lambda), \tilde{Q}_{m-1}(\lambda))$

$$\begin{aligned} 2\tilde{P}_m(\lambda) &= (1 + k_{m-1}) 2^{(m+1)/2} \\ &\quad \cdot \cos\lambda/2 \prod_{i=1}^{\frac{m-1}{2}} (\cos\lambda - \cos\lambda_{m-1,i}) \\ &\quad - (1 - k_{m-1}) 2^{(m+1)/2} \\ &\quad \cdot \sin\lambda/2 \sin\lambda \prod_{i=2}^{\frac{m-1}{2}} (\cos\lambda - \cos\delta_{m-1,i}) \\ &= S_{m,1}(\lambda) - S_{m,2}(\lambda). \end{aligned} \tag{9}$$

The location of the zeros of $\tilde{P}_m(\lambda)$ can be determined according to the following observations:

a) $S_{m,1}(0) = (1 + k_{m-1}) 2^{(m+1)/2} \prod_{i=1}^{\frac{m-1}{2}} (1 - \cos\lambda_{m-1,i}) > 0$; $S_{m,2}(0) = 0$ and $S_{m,2}(\lambda) > 0$ for $\delta_{m-1,1} = 0 < \lambda < \delta_{m-1,2}$.
b) A simple inspection of (9) reveals that polynomials $S_{m,1}(\lambda)$ and $S_{m,2}(\lambda)$ change their sign in passing through a zero at $\lambda_{m-1,i}$ and $\delta_{m-1,i}$, respectively.

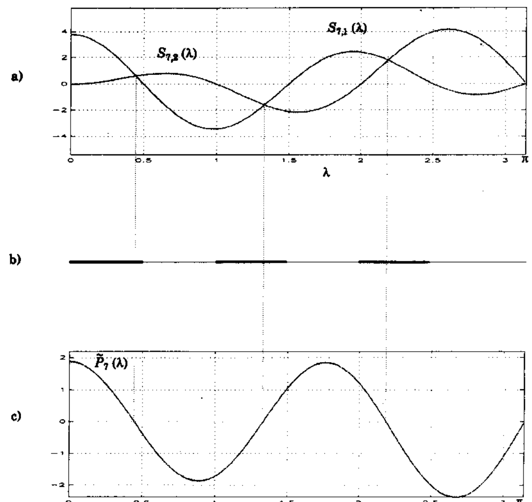

Fig. 2. Example of (a) $S_{7,2}(\lambda)$ and $S_{7,1}(\lambda)$ behavior, (b) location intervals for the roots of $\tilde{P}_7(\lambda) = S_{7,1}(\lambda) - S_{7,2}(\lambda)$, and (c) $\tilde{P}_7(\lambda)$.

Since $\tilde{P}_m(\lambda^*) = 0$ implies sign $[S_{m,1}(\lambda^*)]$ = sign $[S_{m,2}(\lambda^*)]$, properties a) and b) allow one to conclude that $\tilde{P}_m(\lambda)$ takes on a zero value in each of the open $(m-1)/2$ intervals $(\delta_{m-1,i}, \lambda_{m-1,i})$ with $\delta_{m-1,1} = 0$ and $\lambda_{m-1,(m+1)/2} = \pi$. Moreover, there is exactly one zero in each interval because $P_m(z)$ is an m-degree polynomial

It may be pointed out that the actual position of zero $\lambda_{m,i}$ in the interval $(\delta_{m-1,i}, \lambda_{m-1,i})$ depends on the value of the reflection coefficient k_{m-1}, and its shifts from $\lambda_{m-1,i}$ to $\delta_{m-1,i}$ as k_{m-1} varies from $+1$ to -1. As a result, such intervals give the best localization of mth order LSF's based on the values of the LSF's of $(m-1)$th order.

Similarly, it can be verified that the zeros $\delta_{m,i}$ of polynomial $\tilde{Q}_m(\lambda)$ (except $\delta_{m,1} = 0$) belong to the complementary intervals $(\lambda_{m-1,i-1}, \delta_{m-1,i})$, $i \ge 2$. □

Fig. 2(a) shows polynomials $S_{7,2}(\lambda)$ and $S_{7,1}(\lambda)$ corresponding to LSF's frequencies

$$\lambda_{6,1} = 0.5 \quad \lambda_{6,2} = 1.5 \quad \lambda_{6,3} = 2.5$$

$$\delta_{6,1} = 0 \quad \delta_{6,2} = 1.0 \quad \delta_{6,3} = 2.0 \quad \delta_{6,4} = \pi$$

and the reflection coefficient $k_6 = 0.15$. Fig. 2(b) gives the intervals where the zeros of $\tilde{P}_7(\lambda)$ must lie; for $k_6 = 0.15$ its zeros correspond to the intersection points of polynomials $S_{7,2}(\lambda)$ and $S_{7,1}(\lambda)$ as shown in Fig. 2(c).

III. Further Considerations

In this section, we shall show a connection (or a lack of connection) between the localization property of the LSF's frequencies and the separation property of orthogonal polynomial roots.

A fundamental separation theorem regarding orthogonal polynomials deals with the connection between the roots of successive order polynomials [14]. It states that, given the roots $x_1 < x_2 < \cdots < x_m$ ($x_0 = a$, $x_{m+1} = b$) of the polynomial $P_m(x)$, belonging to a set of polynomials orthogonal on the real interval $[a, b]$, then each interval $(x_\nu, x_{\nu+1})$, $\nu = 0, \cdots, m$, contains exactly one root of $P_{m+1}(x)$.

Hence, the localization property shown in the previous section, regarding the zeros of $\tilde{P}_m(\lambda)$ and $\tilde{Q}_m(\lambda)$ in terms of those of $\tilde{Q}_{m-1}(\lambda)$ and $\tilde{P}_{m-1}(\lambda)$, would suggest the existence of a "certain" orthogonality relationship between these polynomials. A result in this direction has been achieved by Delsarte and Genin [7], which proved, via a proper map from the unit circle to the real axis, that $\tilde{P}_m(\lambda)$ and $\tilde{Q}'_{m-1}(\lambda) = \tilde{Q}_m(\lambda)/\sin\lambda/2$ constitute two sets of orthogonal polynomials with different weighting functions. From the previously quoted separation theorem, a first evaluation of the location interval of the roots of $\tilde{P}_m(\lambda)$ and $\tilde{Q}'_{m-1}(\lambda)$ (i.e., the LSF's frequencies of mth order) can be given. Thus, with the notation used along the paper, we can say that if $(\lambda_{m-1,i}, \lambda_{m-1,i+1})$ is the interval between successive LSF's corresponding to $\tilde{P}_{m-1}(\lambda)$ then, there exists exactly one root, $\lambda_{m,i}$, of $\tilde{P}_m(\lambda)$ in that interval. A similar result holds for the zeros of $\tilde{Q}'_{m-1}(\lambda)$. Notice that, in this respect, the result of the previous section uses the dependence of $\tilde{P}_m(\lambda)$ and $\tilde{Q}'_{m-1}(\lambda)$ from both $\tilde{P}_{m-1}(\lambda)$ and $\tilde{Q}'_{m-2}(\lambda)$ to give a more precise localization of each zero.

From these considerations one could expect that such a localization property is closely related to the orthogonality of the two sets: $\{\tilde{P}_0(\lambda), \tilde{Q}'_1(\lambda), \tilde{P}_2(\lambda)\cdots \tilde{P}_{M-1}(\lambda), \tilde{Q}'_M(\lambda), \tilde{P}_{M+1}(\lambda)\}$ and $\{\tilde{Q}'_0(\lambda), \tilde{P}_1(\lambda), \tilde{Q}'_2(\lambda)\cdots \tilde{Q}'_{M-2}(\lambda), \tilde{P}_{M-1}(\lambda), \tilde{Q}'_M(\lambda)\}$. Namely, such a location property would comply with the orthogonality of $\tilde{P}_m(\lambda)$ to both $\tilde{Q}'_{m-1}(\lambda)$ and $\tilde{P}_{m-1}(\lambda)$, whose zeros are known to interlace: each zero $\lambda_{m,i}$, $i \geq 1$, of $\tilde{P}_m(\lambda)$ belongs to the intersection of the intervals $(\lambda_{m-1,i}, \lambda_{m-1,i+1})$, with $\lambda_{m-1,0} = 0$, and $(\delta_{m-1,i}, \delta_{m-1,i+1})$. A similar property would apply to the zeros of $\tilde{Q}'_m(\lambda)$. However, the orthogonality of the two previously defined sets, does not hold, since $\tilde{P}_m(\lambda)$ is orthogonal to $\tilde{Q}'_{m-1}(\lambda)$ through a weighting function different from that associated to the orthogonality of $\tilde{Q}'_{m-1}(\lambda)$ and $P_{m-2}(\lambda)$.

Below, the structure of such polynomials sets is studied, as in [7], via Favard's theorem [15], which states that, a succession of real polynomials $p_n(x)$ ($p_0(x) = 1$, $p_1(x) = x - a_1$) is orthogonal with respect to an increasing function if and only if they are generated via a three-term recursion: $p_n(x) = (x - a_n)p_{n-1}(x) - \lambda_n p_{n-2}(x)$, where $n \geq 2$, $\lambda_n > 0$ and a_n are real numbers.

To this purpose it is useful to introduce two further relationships between the predictors, $A_m(z)$ and the polynomials $P_m(z)$ and $Q_m(z)$. From (2) and (3) one has

$$(1 + k_m)P_m(z) = A_m(z) + \hat{A}_m(z) \quad (10a)$$

$$(1 - k_m)Q_m(z) = A_m(z) - \hat{A}_m(z). \quad (10b)$$

It has been shown in [7], that (2), (3), and (10) imply the following three-term recurrence for the sets of polynomials $\{P_m(z)\}$ and $\{Q'_m(z)\}$

$$P_{m+1}(z) - (1 + z^{-1})P_m(z) + \alpha_m z^{-1}P_{m-1}(z) = 0 \quad (11a)$$

$$Q'_m(z) - (1 + z^{-1})Q'_{m-1}(z) + \alpha^*_m z^{-1}Q'_{m-2}(z) = 0 \quad (11b)$$

with $\alpha_m = (1 + k_{m-1})(1 - k_m) > 0$ and $\alpha^*_m = (1 - k_{m-1})(1 + k_m) > 0$ and $Q'_m(z) = Q_{m+1}/(1 - z^{-1})$ a symmetric polynomial.

The map

$$x = \frac{z^{-1/2} + z^{1/2}}{2} = \cos\lambda/2, \qquad z = e^{j\lambda}, \qquad 0 \leq \lambda \leq 2\pi \quad (12)$$

induces a one-to-one correspondence between the unit circle and real interval $[-1, +1]$. Then defining two polynomials in the real variable x

$$p_m(x) = z^{+m/2}P_m(z) = \tilde{P}_m(z)$$
$$q_m(x) = z^{+m/2}Q'_m(z) = \tilde{Q}'_m(z) \quad (13)$$

(11a) and (11b) transform into

$$p_{m+1}(x) = 2xp_m(x) + \alpha_m p_{m-1}(x) \quad (14a)$$

$$q_m(x) = 2xq_{m-1}(x) + \alpha^*_m q_{m-2}(x). \quad (14b)$$

Equations (14a) and (14b) are the device used to prove, via Favard's theorem [15], the orthogonality of polynomials $p_m(x)$ and $q_m(x)$. Along the same lines, a three-term recurrence between polynomials $p_m(x)$ and $q_m(x)$ can be found. To this purpose, using the x-domain version of (5) and (14), it is

$$q_m - (1 + k_{m-1})xp_{m-1} + \left(\alpha^*_m - 2x^2(1 - k_{m-1})\right)q_{m-2} = 0 \quad (15a)$$

$$(1 - 2x^2)p_{m+1} + 4x(x^2 - 1)q_m + \alpha_m p_{m-1} = 0. \quad (15b)$$

Equations (15) allow one to build up the sets $\{q_{-1}(x), p_0(x), q_1(x)\cdots p_{M-2}(x), q_{M-1}(x), p_M(x)\}$ and $\{q_0(x), p_1(x), q_2(x)\cdots p_{M-1}(x), q_M(x), p_{M+1}(x)\}$ starting from $p_0(x) = 2$, $p_1(x) = 2x$, $q_{-1}(x) = 0$ and $q_0(x) = 1$. The two relationships, apart from computational efficiency, are equivalent to recursions (14). However, since the three-term recurrences (15) do not respect the conditions of Favard's theorem, the corresponding polynomials are not orthogonal. This gives an alternate proof that the localization property of Section 2 is not related to orthogonality.

IV. Conclusion

The main aim of this paper has been to provide a deeper insight in the LSF's properties. It has been proved that there exists a strong relationship between the location of the zeros of $\tilde{P}_m(\lambda)$ and $\tilde{Q}_m(\lambda)$ (i.e. the LSF's of mth order) and the intervals between the zeros of $\tilde{P}_{m-1}(\lambda)$ and $\tilde{Q}_{m-1}(\lambda)$ (i.e. the LSF's of $(m-1)$th order). It results in the most precise lower and upper bounds of intervals which the LSF's may belong to, a property that matches very well with the robust computational procedure presented in [6].

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On the Periodicity of Speech Coded with Linear-Prediction Based Analysis by Synthesis Coders

W. Bastiaan Kleijn

***Abstract*—The closed-loop pitch predictor (CLPP) is an essential part of most linear-prediction based analysis-by-synthesis (LPAS) coders. This correspondence discusses the relation between the periodicity of the original and reconstructed signals for LPAS coders with a CLPP. It is shown that the periodicity is unchanged if the periodicity of the original signal is generated with an autoregressive model and increases or decreases for several other signal classes. The theoretical findings are confirmed by experiments.**

I. Introduction

With few exceptions, recent implementations of linear-prediction based analysis-by-synthesis (LPAS) speech coders, such as multipulse and code-excited linear prediction (CELP), employ a closed-loop pitch predictor (CLPP) [1] to increase the coding efficiency. The determination of the CLPP parameters is an integral part of the analysis-by-synthesis mechanism. The same basic procedure is used by most implementations, although it may vary in details. In this procedure, the CLPP parameters are determined on a block-by-block basis, a block usually being referred to as a subframe. For each subframe, the zero-input response of the CLPP is obtained for all allowed delay values, including noninteger sample delays [2]. Each of the CLPP zero-input responses is filtered with the linear-prediction (synthesis) filter. These "candidate" reconstructed speech signal segments are compared to the original speech signal, using a perceptually-meaningful criterion, and the delay corresponding to the candidate segment which best matches the original is selected. Once the CLPP parameters are found, its excitation is determined, again using analysis-by-synthesis. This determination of the excitation varies between the different LPAS coders.

Although the basic CLPP greatly enhances the coding efficiency of the LPAS coders, the perceived level of periodicity of the reconstructed speech quality tends to decrease with decreasing bit rate. This is associated with a diminished speech quality. To counter this effect, heuristic procedures have been introduced to enhance the periodicity of the reconstructed signal [3]–[8]. Most of these procedures result in a significant decrease of the signal-to-noise ratio and yet increase the speech quality. Both an increase and a decrease of the perceived level of periodicity significantly affect the speech quality. An increase often results in "buzziness" while a decrease often results in a noisy character. In the periodicity-enhancement procedures one usually has to balance these distortions.

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Relatively little work has been done to quantify the properties of the CLPP. The pitch predictor has been analyzed for stability [9], but the efficiency with which the CLPP reconstructs periodicity has not been quantified. The goal of this correspondence is to provide new insight in the relation between the periodicity of the input signal, properties of the input signal, and the periodicity of the output. In Section II, periodicity is defined by a criterion. A relation of the periodicity of the original and the reconstructed speech signals is then derived in a nonrigorous fashion, and its consequences are discussed. Experimental results which confirm this relationship are presented in Section III and conclusions are provided in Section IV.

II. Relation of Periodicity of Original and Coded Speech

A. Constraints

The present work is aimed at the single-tap CLPP, with a parameter update once per subframe. The signal to be analyzed is assumed to be in a steady state and to have a constant pitch period. The spectral weighting (the linear-prediction and perceptual-weighting filters) used in LPAS coders is ignored in the derivations. However, the validity of the derivations is not limited to the case where spectral weighting is not applied. For each subframe a difference exists between the zero-input response of the spectral-weighting filter for the reconstructed signal and the same response for the original signal. To enhance performance, this difference is subtracted from the target vector (the vector to be matched) in the spectrally weighted domain in most (but not all) LPAS coders. The present derivations do not account for this difference in the zero-input responses of the spectral-weighting, but otherwise the results are valid for the spectrally weighted case.

The pitch period is assumed to be longer than the subframe length used for the quantization of the residual signal of the LPAS coder. Note that the adaptive-codebook approach [10] and the conventional filter approach to the CLPP are identical in this case. Furthermore, when the pitch period exceeds the subframe length, the excitation to the linear-prediction filter is simply an addition of a CLPP contribution and a fixed-codebook contribution (this broad interpretation of the fixed codebook includes, e.g., multipulse structures).

The following notation is used. Signals are indicated by parentheses, e.g., $x()$. Their value at a particular time a is indicated as $x(a)$. Vectors describing discrete-signal segments are characterized by their signal begin and endpoints

$$x(a : a+d) = [x(a), x(a+1), x(a+2), \ldots, x(a+d-1)]^T. \quad (1)$$

The superscript ^{T} denotes the vector (or matrix) transpose. The notation $E[\ ]$ denotes the ensemble average of a parameter, and $P(\cdot)$ denotes probability.

B. Definition of a Periodicity Criterion

Let d be the pitch period. Then, the periodicity is defined here as the expectation value of the measure

$$Y(x(), a, d) \equiv \frac{x(a : a+d)^T x(a-d : a)}{x(a-d : a)^T x(a-d : a)}. \quad (2)$$

Although this criterion is not symmetrical in time, it is more amenable to analytic manipulation than the normalized autocorrelation measure. Experimental evaluations of the latter measure, similar to those in Section III, give similar results to the ones described for (2).

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